SM Report 93-31

EFFECTIVE ELASTIC MODULI AND CHARACTERIZATION OF A PARTICULATE-REINFORCED METAL MATRIX COMPOSITE WITH DAMAGED PARTICLES

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Abstract

A brief derivation of the expression is given for the effective bulk modulus of discontinuously reinforced metal matrix composites (DMMCs) with damaged particles (*either* complete voids as shattered particles *or* debonded particles). The analytical results are then compared with elastic moduli determined from nondestructive ultrasonic wave speed measurements of SiC particle-reinforced titanium matrix composites produced via shock wave consolidation. For the shock consolidated Ti-SiC metal matrix composites compacts, the overall particle damage mode is found to be similar to debonded particles and the effective volume fraction of damaged particles is determined to be 39% based on the data of *both* Young's and bulk moduli. Ultrasonic wave speed measurements combining with analytical and/or numerical results on overall elastic properties (Young's, bulk, and shear moduli, and Poisson's ratio) could be a useful tool in assessing the damage of particles in DMMCs.

1. INTRODUCTION

Fracture of reinforcement particles either during processing or during subsequent mechanical loading have been observed experimentally in discontinuously particulate-reinforced metal matrix composites (DMMCs).[1-4] Characterization of damaged particles will be valuable in improving processing procedures and in understanding deformation and failure of DMMCs. The number of fractured particles can be measured using optical and electron microscopy and particle fracture is found to be influenced by the size, shape, and spatial distribution of the reinforcement particles. Even though the *number* of particles damaged in DMMCs is a very useful information, the effect of damaged particles on elastic and other properties of DMMCs is directly related to the *effective volume fraction* of those particles. Indirect methods such as Wiebull statistical model can be used to calculated the effective volume from the number of damaged particles;[4-5] this approach is, however, inherently inaccurate. Surface damages are common during preparation of DMMCs specimens for microscopy examination and some microscopic cracks within particles may be difficult to detect.

Macroscopically, the damage of particles can also be assessed by comparing the overall elastic moduli determined either by calculating the elastic unloading slope in a tensile test[5] or by measuring elastic ultrasonic wave speeds[6] with predictions of existing analytical and/or numerical models. There exists a rich literature of theoretical models in estimating overall elastic properties of DMMCs when reinforcements are assumed to be *perfectly bonded*.[7-8] Analytical results such as the Hashin-Shtrikman bounds,[9] self-consistent estimates,[10] and mean-filed theories[11] are found to be well applicable to DMMCs with randomly distributed particles of low aspect ratio. Finite element analyses are also used to study the overall elastic properties of two-phase DMMCs as a function of the shape, concentration, and spatial distribution of the reinforcement particles.[12] Recently, analytical models of overall elastic moduli of DMMCs with damaged particles have been formulated.[13] By combining Eshelby's equivalent inclusion method and Mori-Tanaka's back stress analysis,[14-15] Mochida *et al.*[13] obtained the analytical expressions of Young's modulus of DMMCs for three particle damage models: the shattering of particles producing complete voids, debonding of at the interface between the matrix and particles, and aligned penny-shaped cracks within particles. A numerical study of Young's modulus of DMMCs with *aligned* penny-shaped cracks within particles has also been reported.[12] However, the actual damage mode of particles cannot be determined through these results on Young's modulus *alone* and thus the *effective volume fraction* of damaged particle remains uncertain.

We propose here that for some DMMCs it is possible to determine *both* the overall damage mode of particles and the effective volume fraction of damaged particles by including the second elastic constant, *the bulk modulus*, in both experimental measurements and analytical modeling. At first, we present a brief derivation of the expression for the effective bulk modulus of DMMCs with damaged particles (*either* complete voids as shattered particles *or* debonded particles) following the approach of Mochida *et al.*[13] The analytical results are then compared with elastic moduli determined from nondestructive ultrasonic wave speed measurements of SiC particle-reinforced titanium matrix composites produced through shock wave consolidation.[16] Based on the data of Young's modulus, the effective volume fraction of damaged particles is estimated to be about 31%, 39%, and 46% for three particle damage modes — complete voids, debonded particles, and particles containing aligned single penny-shaped cracks, respectively. The effective volume fraction of damaged particles is found to be 39% for either complete voids or debonded particles based on the data of bulk modulus. Thus, for the shock consolidated Ti-SiC DMMCs, the overall particle damage mode is similar to debonded particles and the effective volume fraction of damaged particle is determined to be 39%.

2. ANALYTICAL MODELS

Following Taya and Chou[15] of using the combination of Eshelby's equivalent inclusion method[14] and Mori-Tanaka's back stress analysis,[11] Mochida *et al.*[13] obtained the weakened effective Young's modulus of particulate-reinforced DMMCs when some of particle reinforcements are damaged. The composite is modeled as an infinite elastic body which contains an infinite number of ellipsoidal inhomogeneities with two kinds (undamaged and damaged particles). For the three cases of the shattering of particles producing complete voids, debonded particles, and particles containing single penny-shaped cracks (Fig. 1), their results can be summarized as

$$\frac{E_c}{E_m} = \frac{1}{1+\eta_p(1-f_d)f_0+\eta_d f_d f_0}, \tag{1}$$

where E_c and E_m are Young's moduli of the composite and matrix, respectively, f_0 is the volume fraction of total particle reinforcements, f_d is the volume fraction of damaged particles in terms of total particles, and η_p and η_d are functions of f_0, f_d and elastic moduli of matrix and reinforcements (see the detailed expressions given by Mochida *et al.*[13]).

The second elastic constant, the bulk modulus of the hybrid composites, can also obtained by replacing the uniaxial tensile stress field of Mochiba *et al.*[13] with a hydrostatic tensile stress field imposed on the composite. A brief derivation of the bulk modulus is presented here. The overall stiffness of the composite is determined by using the equivalence of the strain energies (see Eq. 10 in Mochida *et al.*[13]),

$$\frac{1}{2}C_{ijkl}^{c\ -1}\sigma_{ij}^{0}\sigma_{kl}^{0}=\frac{1}{2}C_{ijkl}^{m\ -1}\sigma_{ij}^{0}\sigma_{kl}^{0}+\frac{1}{2}f_p\sigma_{ij}^{0}e_{ij}^{p*}+\frac{1}{2}f_v\sigma_{ij}^{0}e_{ij}^{v*}, \tag{2}$$

where $C_{ijkl}^{c\ -1}$ and $C_{ijkl}^{m\ -1}$ are the compliances of the matrix and the composite, respectively; σ_{ij}^{0} is the applied stress, $f_v(=f_d f_0)$ is the volume fraction of damaged particles, $f_p(=f_0-f_d f_0)$ is the volume fraction of undamaged particles, f_0 is the total volume fraction of reinforcement particles (known for a given DMMC) and f_d is the part of those particles damaged (in terms of volume fraction). The details of the derivation of Eq. (2) are given by Taya and Chou[15]. e_{kl}^{p*} and e_{kl}^{v*} in Eq. (2) are eigenstrains due to the existence of undamaged and damaged particles in the matrix which can be determined through equivalent inclusion method (see the details of derivation of Esq. 1-9 in Mochida *et al.*[13]),

$$C_{ijkl}^{m}\left(e_{kl}^{0}+\overline{e}_{kl}+e_{kl}^{p}-e_{kl}^{p*}\right)=C_{ijkl}^{p}\left(e_{kl}^{0}+\overline{e}_{kl}+e_{kl}^{p}\right), \tag{3}$$

$$C_{ijkl}^{m}\left(e_{kl}^{0}+\overline{e}_{kl}+e_{kl}^{v}-e_{kl}^{v*}\right)=0, \tag{4}$$

$$\left(1-f_p-f_v\right)C_{ijkl}^{m}\overline{e}_{kl}+f_pC_{ijkl}^{m}\left(\overline{e}_{kl}+e_{kl}^{p}-e_{kl}^{p*}\right)+f_vC_{ijkl}^{m}\left(\overline{e}_{kl}+e_{kl}^{v}-e_{kl}^{v*}\right)=0, \tag{5}$$

$$\text{and } \sigma_{ij}^{0}=C_{ijkl}^{m}e_{kl}^{0},\quad e_{kl}^{p}=S_{klmn}^{p}e_{mn}^{p*},\quad e_{kl}^{v}=S_{klmn}^{v}e_{mn}^{v*}, \tag{6}$$

where C_{ijkl}^{p} is the stiffness of the reinforcement particle, e_{kl}^{0} is the strain disturbance in the matrix without any reinforcements, $\overline{e}_{kl}$ is the averaged strain disturbance in the matrix due to all undamaged and damaged reinforcement particles, e_{kl}^{p} is the strain disturbance in the undamaged particle, e_{kl}^{v} is the strain disturbance in the damaged particle, S_{klmn}^{p} is Eshelby's tensor for the undamaged particle which depends on C_{ijkl}^{m} and the geometry of the particle, and S_{klmn}^{v} is Eshelby's tensor for the damaged particle. If damaged particles are represented as spherical voids or debonded particles, $S_{klmn}^{p}=S_{klmn}^{v}$.

For a hydrostatic stress field,

$$\sigma_{11}^{0}=\sigma_{22}^{0}=\sigma_{33}^{0}=\sigma_0,\ \text{all other } \sigma_{ij}^{0}=0, \tag{7}$$

Eq. (2) is reduced to

$$\frac{\sigma_0^2}{K_c} = \frac{\sigma_0^2}{K_m} + 3f_p\sigma_0 e^{p*} + 3f_v\sigma_0 e^{v*}, \tag{8}$$

where K_c and K_m are bulk moduli of the composite and matrix, respectively, and Eq. (6) is reduced to

$$\sigma_0 = 3K_m e_0, \quad e^p = (S_1 + 2S_2)e^{p*}, \quad e^v = (S_1 + 2S_2)e^{v*}. \tag{9}$$

where for spherical voids or debonded particles, we have (see Appendix A in Mochida *et al.*[13])

$$S_1 = S^p_{1111} = \frac{7-5\nu_m}{15(1-\nu_m)}, \quad S_2 = S^p_{1122} = -\frac{1-5\nu_m}{15(1-\nu_m)}, \tag{10}$$

where ν_m is the Poisson's ratio of the matrix. Subsequently, Eqs. (3-5) can be reduced to

$$\sigma_0 + 3K_m\bar{e} + 3K_m(S_1 + 2S_2 - 1)e^{p*} = \frac{K_p}{K_m}\sigma_0 + 3K_p\bar{e} + 3K_p(S_1 + 2S_2)e^{p*}, \tag{11}$$

$$\sigma_0 + 3K_m\bar{e} + 3K_m(S_1 + 2S_2 - 1)e^{v*} = 0, \tag{12}$$

$$\bar{e} + f_p(S_1 + 2S_2 - 1)e^{p*} + f_v(S_1 + 2S_2 - 1)e^{v*} = 0, \tag{13}$$

where K_p is the bulk modulus of the reinforcement.

The bulk modulus of the composite can then be obtained by solving the eigenstrains in Eqs. (11-13) and by substituting them into Eq. (8). The final result for the bulk modulus is given as

$$\frac{K_c}{K_m} = \frac{1}{1 + 3\eta_k(1-f_d)f_0 + 3\eta_v f_d f_0}, \tag{14}$$

where η_k and η_v are functions of f_0, f_d, and elastic constants of matrix and reinforcements, namely,

$$\eta_k = \frac{1}{3(1-f_d)f_0(S-1) + 3(f_d f_0 - 1)\dfrac{K_p S - K_m(S-1)}{K_p - K_m}}, \tag{15}$$

$$\eta_v = \eta_k \frac{K_p S - K_m (S-1)}{(K_p - K_m)(S-1)}, \tag{16}$$

$$S = S_1 + 2S_2 = \frac{1+\nu_m}{3(1-\nu_m)}. \tag{17}$$

3. APPLICATION OF ANALYTICAL RESULTS AND DISCUSSIONS

Due to their excellent high temperature properties, Ti-SiC metal matrix composites have been considered to be a potential structural material in aerospace industry.[17] These materials are generally difficult to produce using traditional diffusion bonding or hot-press methods because of the high reactivity of Ti with most reinforcement materials.[18] Recently, Ti-SiC particulate reinforced metal matrix composites have thus been processed by shock wave consolidation.[2,16] The extremely high pressure and short process time (on the order of a microsecond) achieved during shock consolidation have made it feasible to obtain fully dense compacts with *a minimum of interfacial reactions*. Subsequent heat treatments of the composites can produce controlled interfacial reaction zone sizes. The processed materials are ideal for studying effects of interfacial properties on the mechanical behavior of particulate reinforced metal matrix composites.

Particulate reinforced Ti-SiC metal matrix composites were fabricated with the Keck Dynamic Consolidation Facility. The facility is built around a 3 meter long and 35 mm in diameter smooth bore cannon barrel supplied through the courtesy of Aerojet Ordinance. The propellant cannon was designed specifically for one-dimensional plane shock wave consolidation and recovery of materials.[19] Three types of titanium-SiC composite powders were used: Type I and Type II titanium-SiC composite powders were obtained by blending the corresponding pure titanium powder (particle sizes of -100 mesh and -325 mesh respectively) with 10% volume fraction of #600 SiC powder with an average particle size of 12 μm (Electro-Abrasive, Inc.). Type III composite powder was obtained by ball-milling of the -325 mesh titanium powder plus 10% vol. #600 SiC powder for 9 hours (resulting the average size of SiC is about 1-2 μm). Before consolidation, Types I and II powders were degassed for six hours and Type III powder was degassed for twelve hours at 600°C in a vacuum of 5×10^{-6} Torr. The degassed powder was transferred directly into an argon-filled glovebox without being exposed to the laboratory atmosphere. Packing of powder into the target fixture was conducted in the sealed glovebox and the powder was sealed with a thin aluminum foil at the top of the target. The thin aluminum foil reduces oxygen and other gas contaminations during installation of the target into the compactor chamber. After flushing the gun barrel with argon gas, the gun barrel was pumped to a vacuum of about 25 mTorr causing the thin aluminum foil to break off and allow proper degassing of packed powder before shock consolidation. A 31.5 mm diameter flyer plate is mounted onto a nylon sabot and accelerated by smokeless shotgun or pistol powder. Flyer speeds of up to 2 km/s can be obtained. The densification of the powders is accomplished by the passage of a strong shock wave generated upon the impact of flyer onto the green compact (target powder).

Fully dense composite compacts (31-32 mm in diameter and 7-8 mm in height) have been obtained that are free from interfacial reactions and *macroscopic* cracks (Fig. 2a). Assuming Young's modulus E=450 GPa and 110 GPa, Poisson's ratio υ=0.17 and 0.324 for SiC particles and titanium matrix respectively, the Hashin-Shtrikman bounds[9] for both Young's modulus and bulk modulus of 10% vol. undamaged SiC particle reinforced titanium matrix composites are 124.8 GPa $\leq E \leq$ 133.1 GPa, 111.3 GPa $\leq K \leq$ 113.6 GPa. Self-consistent estimates[10] are E=125.3 GPa and K=111.4 GPa. Mori-Tanaka's mean field theory[11] gives E=124.9 GPa and K=111.4 GPa. Figs. 3 (a) and (b) show results of Young's and bulk moduli of Ti-SiC composites by Hashin-Shtrikman variational method, self-consistent scheme, and Mori-Tanaka mean field theory for different volume fraction of reinforcements. Results of elastic moduli of composite compacts #151 and #152 are also shown in Figs. 3 (a) and (b) based on measurements of ultrasonic wave speeds.[16] The composite compacts #151 and #152 were made of Type I and Type II titanium-SiC powders respectively.

The existence of some microscopic defects in composite compacts #151 and #152 can be indirectly deduced from the much lower-than-expected effective overall elastic moduli of these composites (both the Young's modulus and bulk modulus of compacts #151 and #152 are outside the Hashin-Shtrikman lower bounds). The micrographs of these composites show some fracture and damage of large SiC particles (Fig. 2b). Also, the bonding between SiC particles and surrounding titanium matrix may not be perfect as well (note nearly zero interfacial reaction). It is clear that fracture of the SiC particles occurs during shock consolidation because of the high pressures present and/or thermal contraction mismatch upon subsequent cooling to room temperature. In addition, the SiC particles have irregular shapes and may have some preexisting internal defects. Number fractions of cracked particles in shock consolidated composites compacts were reported to range from 90% for a large average SiC particle size of 104 μm to 5% for a small average SiC particle size of 12 μm.[2] SiC particles with a small average size of 12 μm are used in composite compacts #151 and #152.

An estimate of the *volume* fraction of damaged particles in composite compacts can be made by applying the analytical results for Young's modulus of DMMCs by Mochida *et al.*[13] and for bulk modulus given above (Eqs. 14-17). The composite is assumed to consist of titanium matrix, perfectly bonded spherical SiC particles, and spherical voids *or* debonded SiC particles (replacing fractured SiC particles). The Young's and bulk moduli of such composites can be computed for a given volume fraction of perfectly bonded SiC particles and spherical voids or debonded SiC particles. The results for 10% vol. SiC particulate reinforced titanium composites are shown in Fig. 4. Even though only about 5-10 % of SiC particles were microscopically observed to be apparently fractured in the as-consolidated compacts #151 and #152, the *effective* volume fraction of damaged particles can be much higher because larger particles are more prone to fracture and a lot of microcracks may exist. From the plots shown in Fig. 4, the model of *debonded SiC particles* is found to be a fairly good representation of damaged SiC particles and the estimated volume fraction of damaged SiC particles is about 39% in compacts #151 and #152. For compact #154 made of Type III titanium-SiC powder with a much smaller average size of SiC particles (1-2 μm), its Young's modulus is within the Hashin-Shtrikman bounds and its bulk modulus is very close to the lower bound which indicated that smaller SiC particle reinforcements (1-2 μm) would cause much less or virtually no cracking of particles.[16]

In summary, large particles were found prone to crack in shock consolidated Ti-SiC DMMCs. While only about up to 10% of particles were observed microscopically damaged in composite compacts, the effective volume fraction of damaged particles was estimated to be about 39%. Cracked particles have effects on elastic moduli similar to that of debonded particles, namely, there is still some constrains on uniaxial deformation of the matrix imposed by those damaged particles. Ultrasonic wave speed measurements combining with analytical and/or numerical results on overall elastic properties (Young's, bulk and shear moduli and Poisson's ratio) could be a useful tool in assessing the damage of particles in DMMCs.

Acknowledgment—The authors would like to acknowledge the support of this research by the Division of Materials Research of the National Science Foundation through grant #DMR-9116570. G.R. acknowledges the support of an NSF Presidential Young Investigator award, grant #MSS-9157846.

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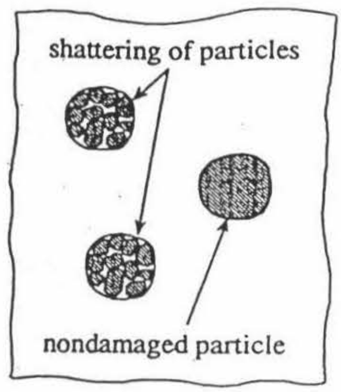
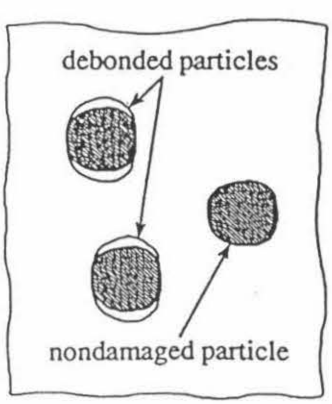
LIST OF FIGURES

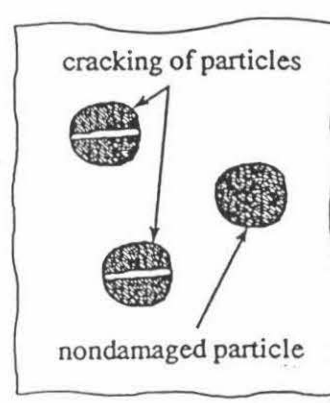
Figure 1



(a) shattered particles
producing complete voids.



(b) particles with the debonded interface
producing debonded particles.



(c) cracked particles producing
penny-shaped cracks within the particles.

Figure 2 (a)

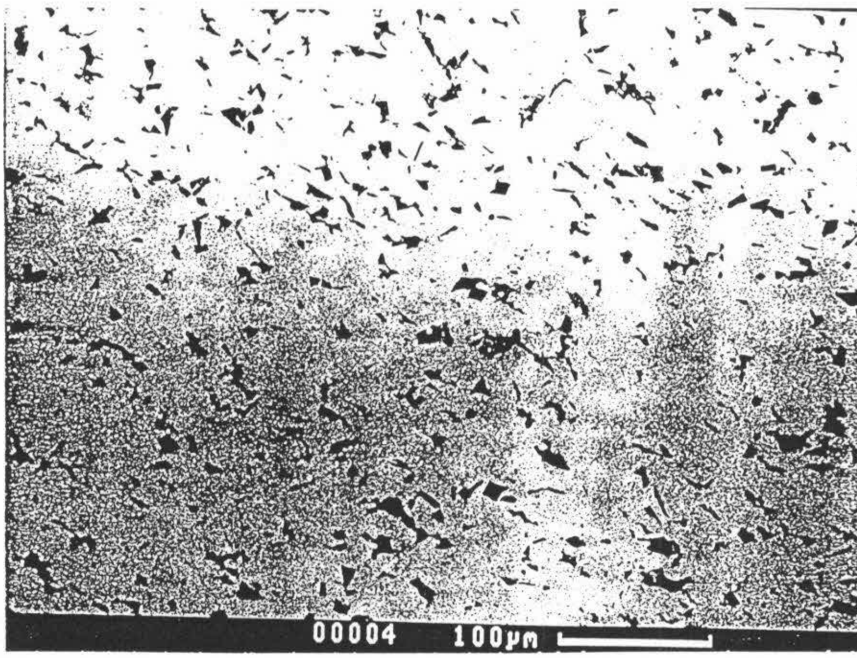


Figure 2 (b)

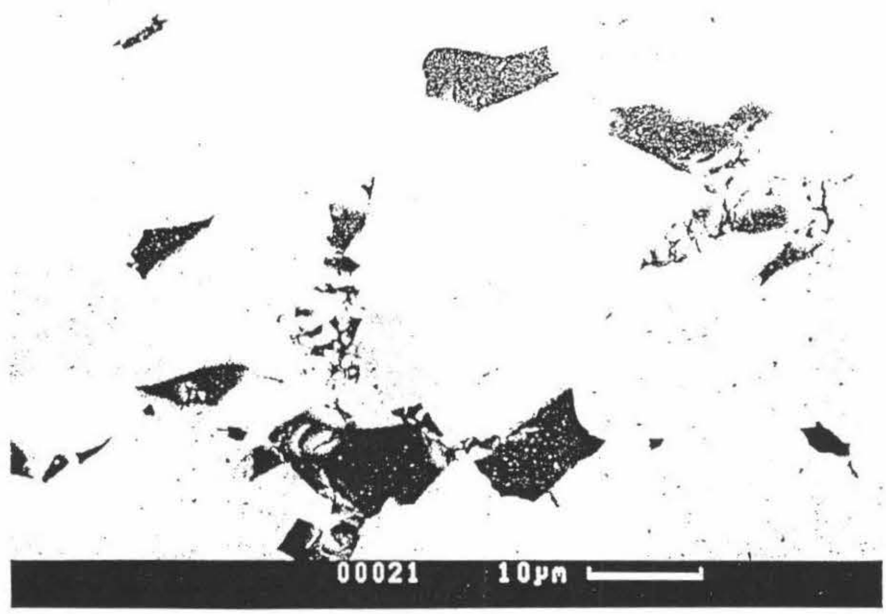
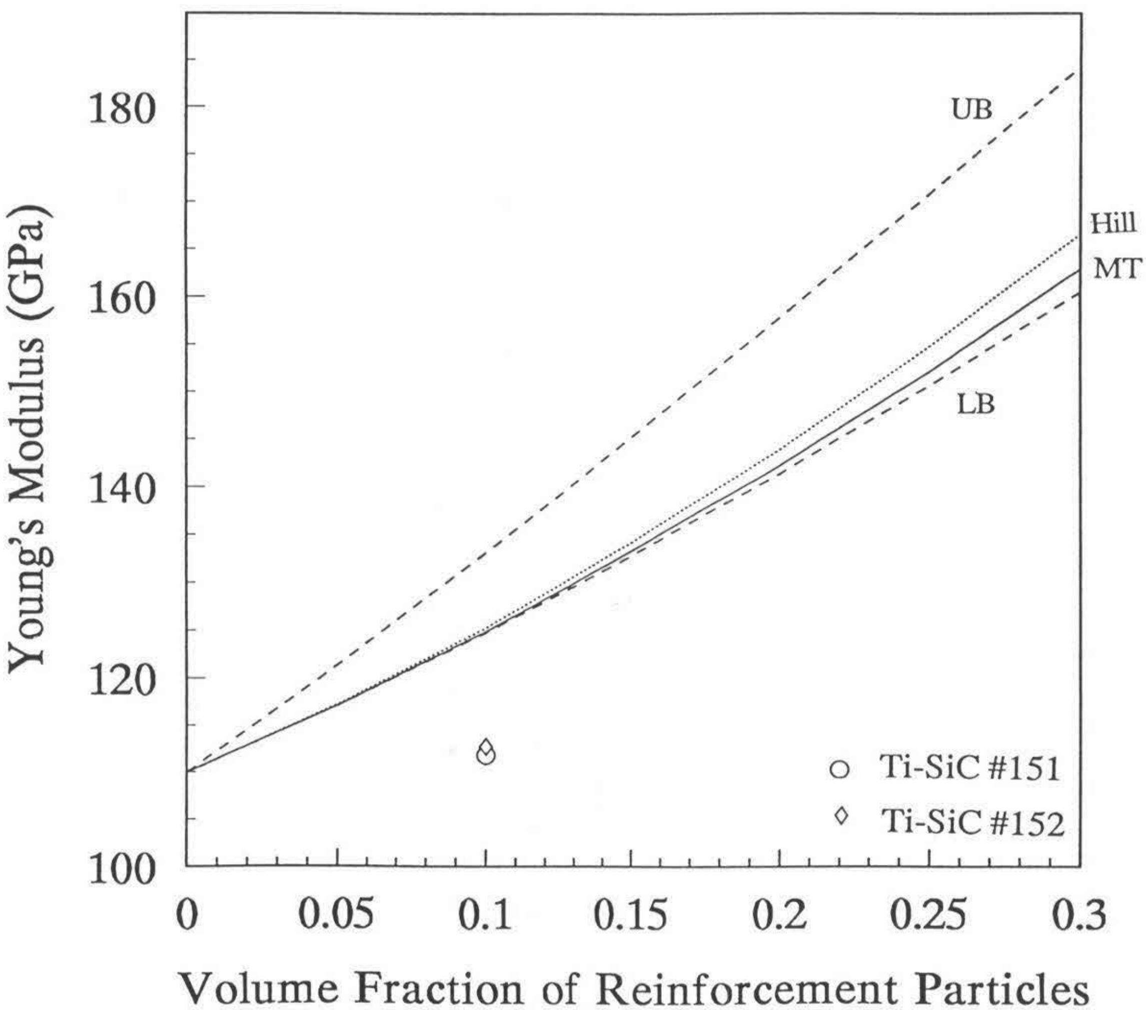
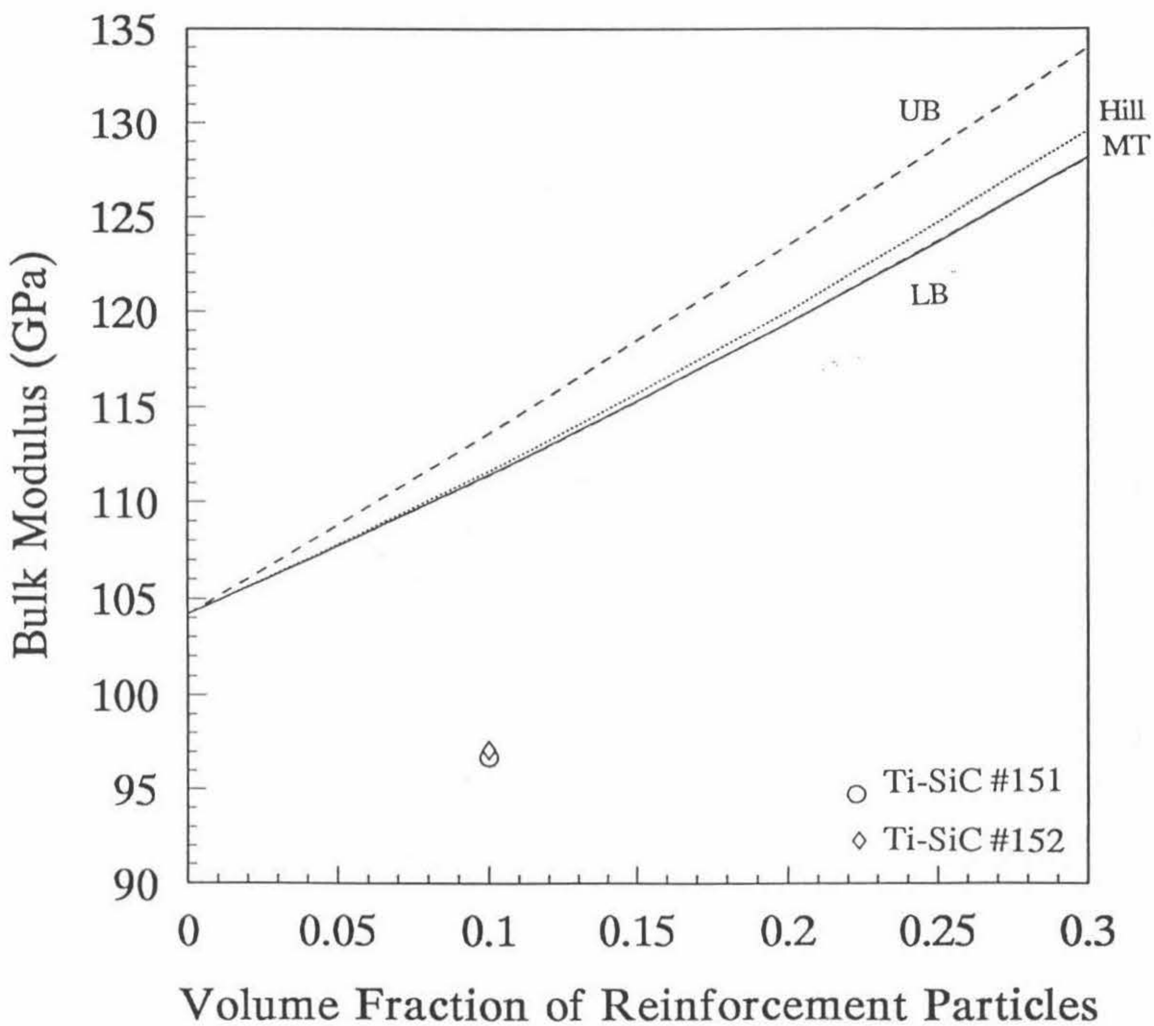


Figure 3 (a)



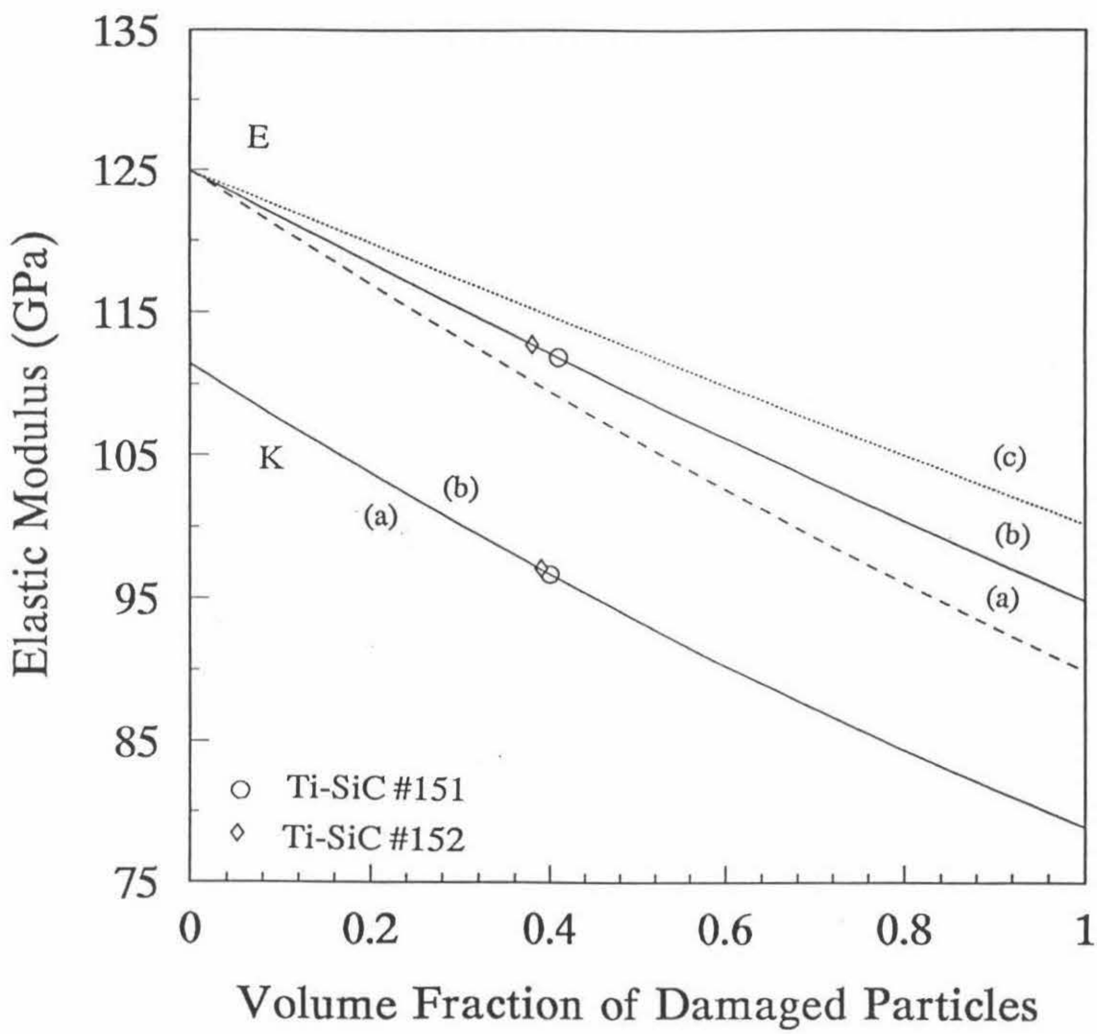
180
160
140
120
100
Young's Modulus (GPa)
UB
Hill
MT
LB
Ti-SiC #151
Ti-SiC #152
0
0.05
0.1
0.15
0.2
0.25
0.3
Volume Fraction of Reinforcement Particles

Figure 3 (b)



135
130
125
120
115
110
105
100
95
90
Bulk Modulus (GPa)
UB
Hill
MT
LB
○ Ti-SiC #151
◇ Ti-SiC #152
0
0.05
0.1
0.15
0.2
0.25
0.3
Volume Fraction of Reinforcement Particles

Figure 4



135
125
115
105
95
85
75
E
K
(a)
(b)
(c)
(b)
(a)
○ Ti-SiC #151
◇ Ti-SiC #152
Elastic Modulus (GPa)
0
0.2
0.4
0.6
0.8
1
Volume Fraction of Damaged Particles